Introduction to Theoretical Ecology Assignment 4

Ricker Logistic Growth Model

One unrealistic feature of the discrete logistic growth equation is that N_{t+1} will become negative when $N_t >> K$. An alternative approach is to follow the Ricker logistic equation (Ricker, 1952), a well-known model in fisheries:

$$N_{t+1} = N_t e^{r\left(1-\frac{N_t}{K}\right)}$$

1. Show analytically the equilibrium points and determine their stability criteria. Compare the stability criteria for this model to those for the standard discrete logistic model. (6 pts)

Solution:

(1) Find the equilibrium points:

$$N_{t+1} = N_t = N^*$$

$$N^* = N^* e^{r\left(1-\frac{N^*}{K}\right)}$$

$$N^* = 0, K$$

(2) Analyze the stability by taking the derivative of the right hand side with respect to *N* and evaluate it at the equilibrium points:

$$\frac{\partial f(N)}{\partial N} = \left(1 - \frac{r}{K}N\right) e^{r\left(1-\frac{N}{K}\right)} \quad (r, K > 0)$$

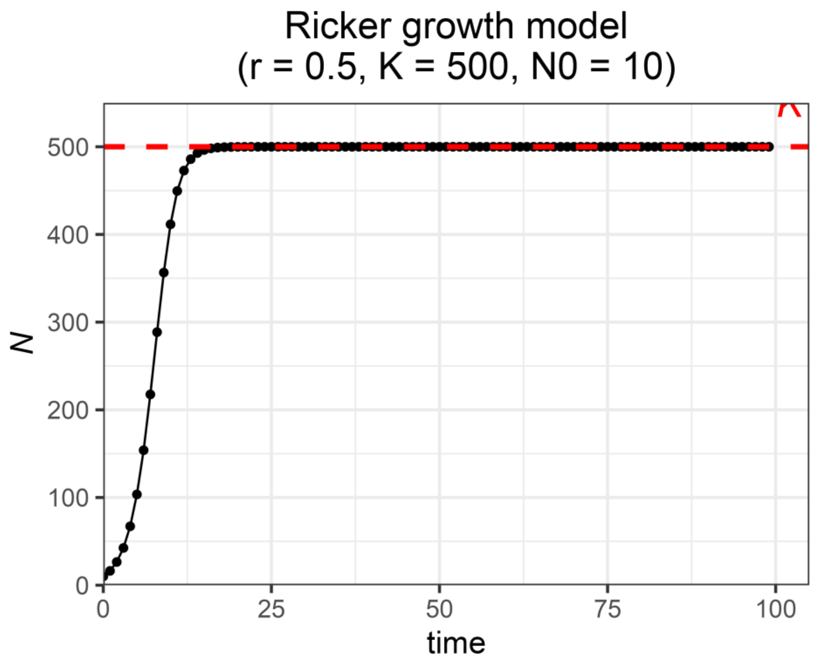
- $N^* = 0$: $\frac{\partial f(N)}{\partial N}|_{N=N^*} = e^r > 1$: unstable; the population will monotonically divert away from the equilibrium
- $N^* = K$: $\frac{\partial f(N)}{\partial N}|_{N=N^*} = 1 - r$
    a. If $1 > r > 0,\ 1 > 1 - r > 0$: K is stable; the population will approach the equilibrium monotonically
    b. If $2 > r > 1,\ 0 > 1 - r > -1$: K is stable; the population will approach the equilibrium with damped oscillations
    c. If $r > 2,\ 1 - r < -1$: K is unstable; the population will oscillate around the equilibrium but never approach it

(3) The stability criteria for Ricker model are the same as those for the standard discrete logistic model ($N^* = 0$ is unstable and $N^* = K$ is stable when $0 < r < 2$). This suggests that Ricker model is probably a better alternative to the discrete logistic model since it avoids the possibility of getting negative population size.

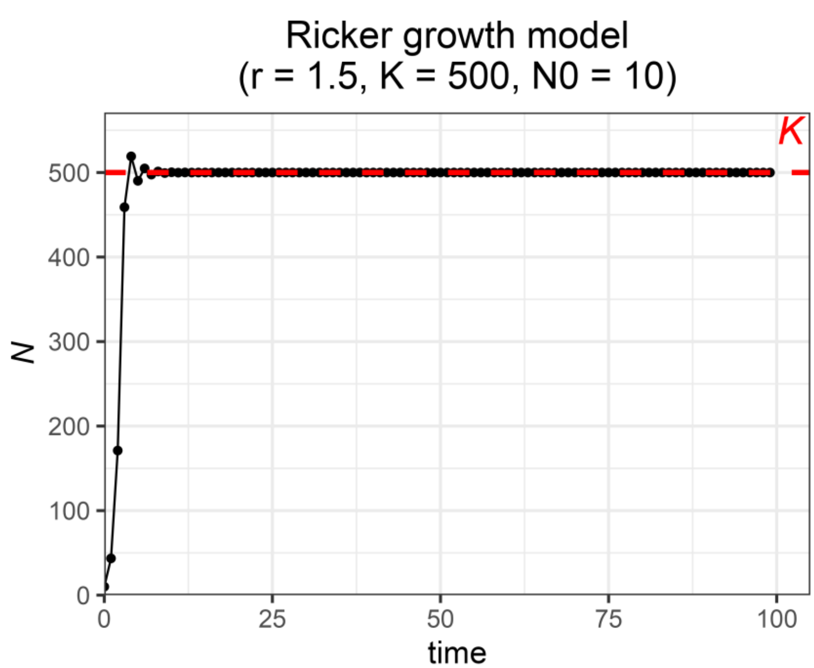
2. Plot the population trajectories under three growth scenarios $r = 0.5$, $r = 1.5$, and $r = 2.7$ ($N_0 = 10$, $K = 500$, 100 time steps for each simulation). Please include the R code you used to generate the results. (4 pts)

Solution:

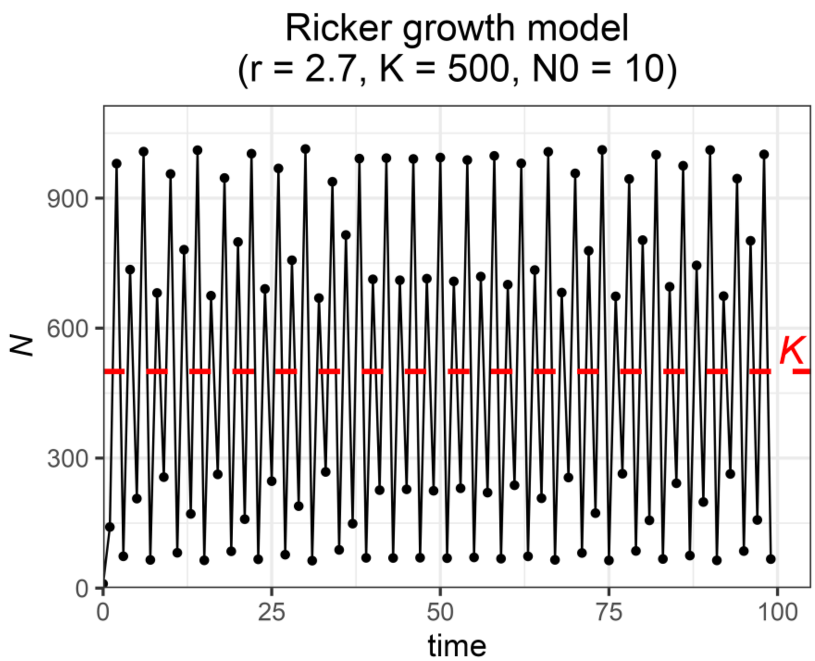
(1) r = 0.5: monotonically approaching the carrying capacity K



(2) r = 1.5: damped oscillations towards the carrying capacity K



(3) *r* = 2.7: bounded oscillations (chaos)



R Code

```
library(tidyverse)

Ricker <- function(r){

  # Set the parameters
  r <- r
  K <- 500
  N0 <- 10
  time <- 100

  # Ricker logistic growth equation
  log_fun <- function(r, N, K){N*exp(r*(1-N/K))}

  # for loop
  pop_size <- numeric(time)
  pop_size[1] <- N0
```

```
  for (i in 2:time) {pop_size[i] <- log_fun(r = r, N = pop_size[i - 1],
K = K)}

  pop_data <- pop_size %>%
    as.data.frame() %>%
    rename(., pop_size = `.`) %>%
    mutate(time = 0:(time-1)) %>%
    relocate(time)

  head(pop_data)

  # Population trajectory
  ggplot(pop_data, aes(x = time, y = pop_size)) +
    geom_point() +
    geom_line() +
    geom_hline(yintercept = K, color = "red", size = 1.2, linetype =
"dashed") +
    geom_text(x = time*1.02, y = K+50, label = "italic(K)", color =
"red", size = 6.5, parse = T) +
    labs(y = expression(italic(N)), title = paste0("Discrete logistic
growth", "\n", "(r = ", r, ", K = ", K, ", N0 = ", N0, ")")) +
    scale_x_continuous(limits = c(0, time*1.05), expand = c(0, 0)) +
    scale_y_continuous(limits = c(0, max(pop_size)*1.1), expand = c(0,
0)) +
    theme_bw(base_size = 15) +
    theme(plot.title = element_text(hjust = 0.5))
}

Ricker(r = 0.5)
ggsave("r0.5.tiff", width = 5.5, height = 4.5, dpi = 600, device =
"tiff")

Ricker(r = 1.5)
ggsave("r1.5.tiff", width = 5.5, height = 4.5, dpi = 600, device =
"tiff")

Ricker(r = 2.7)
ggsave("r2.7.tiff", width = 5.5, height = 4.5, dpi = 600, device =
"tiff")
```