Introduction to Theoretical Ecology Assignment 2

Exponential Population Growth with Constant Immigration

1. You are a curious student in the Introduction to Theoretical Ecology course. After the class, you decide to do a small experiment on population growth. You set up a "massive" fish tank and introduce N_0 flatworm individuals. Also, each day you add I new individuals into the tank, hoping that the population will increase faster. Assuming that the intrinsic rate of increase is r (per day) and there is no factor limiting the growth and reproduction of these flatworms, the population dynamics can be described by the following differential equation:

   $$\frac{dN}{dt} = rN + I$$

   The analytical solution to this differential equation is:

   $$N = N_0 e^{rt} + (e^{rt} - 1)\frac{I}{r}$$

   Please use what you have learned in the lecture to derive the solution for this differential equation step by step. (You can either write down the answer on a paper and embed a picture of it or directly type the equations in Word.) (4 pts)

Solution:

Step 1. Rewrite RHS of the equation: $\frac{dN}{dt} = rN + I = r(N + \frac{I}{r})$

Step 2. Rearrange the terms to separate the variables: $\frac{dN}{(N+\frac{I}{r})} = rdt$

Step 3. Integrate both sides: $\int \frac{dN}{\left(N+\frac{I}{r}\right)} = \int rdt, \ln\left(N + \frac{I}{r}\right) = rt + C_1$

Step 4. Take exponential of both sides: $N + \frac{I}{r} = e^{rt+C_1} = e^{rt}C_2$

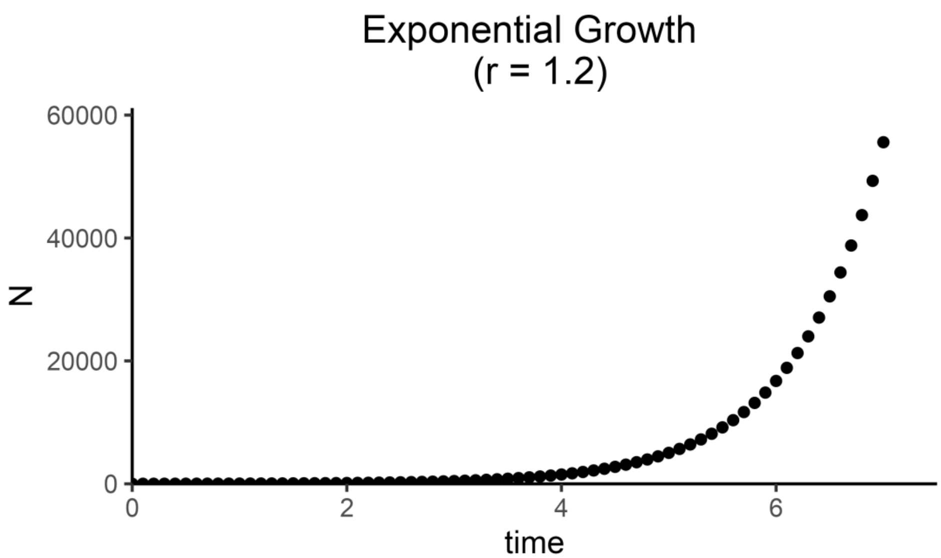
Step 5. Plug in the initial values: $N_0 + \frac{I}{r} = e^0 C_2, C_2 = N_0 + \frac{I}{r}$

Step 6. Substitute C_2 into the equation in step 4: $N + \frac{I}{r} = e^{rt}(N_0 + \frac{I}{r})$

Step 7. Rearrange: $N = e^{rt}\left(N_0 + \frac{I}{r}\right) - \frac{I}{r} = N_0 e^{rt} + (e^{rt} - 1)\frac{I}{r}$

2. Suppose that N_0 = 10, r = 1.2, and I = 3, how will the flatworm population change over a week? Solve the differential equation numerically and visualize the population trajectory. Please show the figure along with the R code you used to generate the results. (You can use any R graphic system you like for plotting). (2 pts)

Solution:



R code:

```
library(deSolve)

library(tidyverse)


exponential_model <- function(times, state, parms) {

  with(as.list(c(state, parms)), {

    dN_dt = r*N + I
```

```
    return(list(c(dN_dt)))  })
}
times <- seq(0, 7, by = 0.1)
state <- c(N = 10)
parms <- c(r = 1.2, I = 3)

pop_size <- ode(func = exponential_model, times = times, y = state,
parms = parms)

ggplot(data = as.data.frame(pop_size), aes(x = time, y = N)) +
  geom_point() +
  labs(title = paste0("Exponential Growth \n (r = ", parms["r"], ")"))
+
  theme_classic(base_size = 12) +
  theme(plot.title = element_text(hjust = 0.5)) +
  scale_x_continuous(limits = c(0,7.5), expand = c(0, 0)) +
  scale_y_continuous(limits = c(0, max(as.data.frame(pop_size)$N)*1.1),
expand = c(0, 0))
```

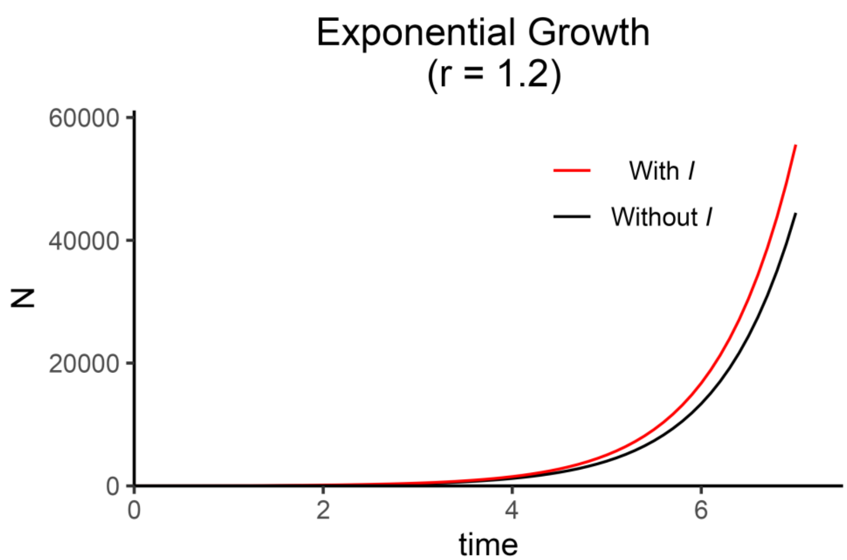
3. Compare the population growth with and without constant immigration and explain the model dynamics in your own words. How does the constant immigration term *I* affect population dynamics? Do you think your daily addition of new flatworm individuals make a big difference? (4 pts)

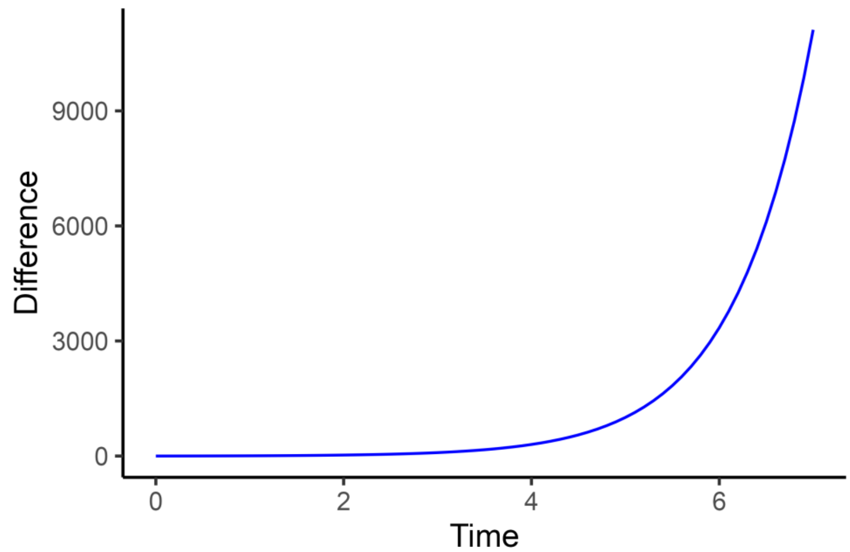
Solution:

Population growth with *I*: $N = N_0e^{rt} + (e^{rt} - 1)\frac{I}{r}$

Population growth without *I*: $N = N_0e^{rt}$

As you can see, the population growth with *I* is actually the population growth without *I* plus another exponential term. So adding a constant *I* to the original differential equation will in fact lead to an exponential addition of individuals.





Also, the ratio of the population size with *I* to the population size without *I* increases in the beginning and level offs over time.